Visualisation of multinomial multilevel time-series modelling

Alice Richardson, Sumonkanti Das, and Bernard Baffour

Australian National University

alice.richardson@anu.edu.au

Research Goal

- Survey data are reliable at the national level, but detailed sub-national estimates are often unreliable due to low or zero sample sizes.
- Small area estimation (SAE) is a statistical technique for producing reliable, detailed-level statistics when data are limited.
- We illustrate the approach using antenatal care (ANC) coverage in Bangladesh, categorizing pregnant women by 0, 1–3, or 4^+ ANC visits, with WHO recommending at least four visits.
- A multinomial multilevel model captures the probabilities of each ANC category across districts and years, accounting for both spatial and temporal correlations.
- Incorporating covariates such as night-time light intensity improves prediction and highlights local disparities over time.
- A spatio-temporal SAE model is applied to estimate district-level ANC trends from 1994–2022, demonstrating the flexibility of multinomial multilevel time-series modelling.

Parameter Estimates

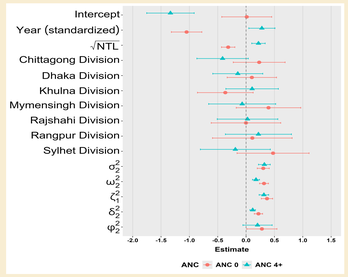

Fig. 2: Fixed (log odds scale) and random effects for ANC0 and ANC4$^+$ under the multinomial multilevel model. Variance components correspond to district- and division-level temporal, spatial, and year-district random effects.

Level	Component 1	Component 2	Mean	t-value	$\hat{R}$
District	Int.: $ANC0$	Int.: $ANC4^+$	-0.709	**-5.519**	1.007
Dist	Int.: $ANC0$	Slope: $ANC0$	0.360	**2.198**	1.000
Dist	Int.: $ANC4^+$	Slope: $ANC0$	-0.155	-0.855	1.000
Dist	Int.: $ANC0$	Slope: $ANC4^+$	-0.194	-0.829	1.000
Dist	Int.: $ANC4^+$	Slope: $ANC4^+$	-0.004	-0.019	1.000
Dist	Slope: $ANC0$	Slope: $ANC4^+$	-0.497	**-2.476**	1.000
Div	RW1: $ANC0$	RW1: $ANC4^+$	-0.297	-1.722	1.003
Dist	RW2: $ANC0$	RW2: $ANC4^+$	-0.796	**-8.476**	1.022

Tab. 1: Correlation between random effects under different random effects components defined for the two outcome variables $ANC0$ and $ANC4^+$ under the multinomial multilevel model. Posterior means, t-values, and Gelman–Rubin $\hat{R}$ statistics are reported. Statistical significance (in **bold**) is indicated by $|t| > 1.96$, and convergence of MCMC chains is assumed confirmed when $\hat{R} < 1.1$.

Model Diagnostics

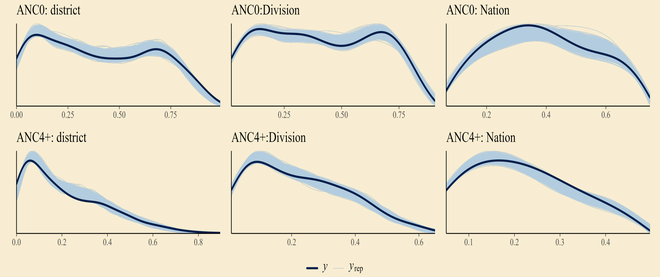

Fig. 5: Posterior predictive check (PPC) comparing model-based simulated draws (thin lines) with kernel densities of direct estimates (thick lines) at district, division, and national levels. The model smooths noisy district-level trends while remaining consistent with higher-level direct estimates.

ANC Coverage and Nightlights

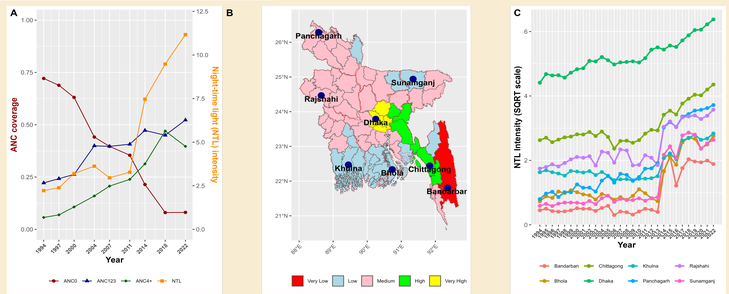

Fig. 1: Country-level trends in ANC coverage and night-time light (NTL) intensity (**A**); district-level NTL intensity in 2024 (**B**); and temporal trends for selected districts (**C**) in Bangladesh. The country comprises 8 divisions (admin-1) and 64 districts (admin-2).

National Trends

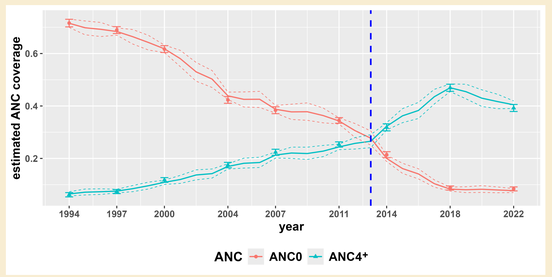

Fig. 3: National level trends in the coverage of 0 and 4^+ ANC visits for pregnant women in Bangladesh during 1994–2022. Solid line shows mean of posterior distribution and 95% credible interval. Direct estimates and 95% confidence intervals shown by dots and whiskers.

Conclusion

- The **multinomial multilevel time-series model** enables the joint analysis of multiple correlated outcomes ($ANC0$, $ANC123$, and $ANC4^+$), capturing dependencies across outcomes.
- Its hierarchical structure borrows strength across space, time, and correlated outcomes, producing stable estimates even for data-sparse districts.
- Model-based estimates maintain numerical consistency across aggregation levels and offer greater precision compared to direct survey estimates.
- Visualising ANC trends within a multinomial framework enhances interpretation of complex spatio-temporal patterns in maternal healthcare access.
- The Bangladesh ANC case study highlights districts exhibiting progress, stagnation, or decline in coverage, with patterns shaped by socioeconomic gradients proxied through night-time light intensity.

Model

- The multinomial multilevel model was specified for counts $\hat{Y}_{it}^{(k)}$ for domains $i = 1$ to 64, years $t = 1994$ to 2022, and outcome categories $k = 1, 2, 3$ corresponding to $ANC0$, $ANC123$, and $ANC4^+$, with category $k = 2$ chosen as the reference. The model can be expressed in its simplest form for domain i as

$$\hat{\boldsymbol{Y}}_{it} \sim \text{Multinomial}(\boldsymbol{\eta}_{it}, \boldsymbol{P}_{it}), P_{itk} = \frac{e^{\eta_{itk}}}{\sum_{k'=1}^{K} e^{\eta_{itk'}}}, \eta_{itk} = \boldsymbol{X}_{it}\boldsymbol{\beta}_k + \sum_{\alpha} \boldsymbol{Z}_{it}^{\alpha}\boldsymbol{v}^{\alpha}$$

$$\log\left(\frac{p_{it}^{(1)}}{p_{it}^{(k)}}\right) = \boldsymbol{\beta}'\boldsymbol{X}_{it} + \nu_i + \nu_i^{(yr)} + u_{it}^{(Div)} + u_{it}^{(Dist)} + s_i$$

  - **Random intercepts** at district-year level: $\nu_i \sim N(0, \sigma_2^2)$,
  - **Random slopes** (time trends) at district level: $\nu_i^{(yr)} \sim N(0, \omega_2^2)$,
  - **Temporal effects** at division level $u_{it}^{(Div)} \sim N(0, \zeta_1^2)$ & at district level $u_{it}^{(Dist)} \sim N(0, \delta_2^2)$,
  - **Spatial effects** at district level: $s_i \sim N(0, \varphi_2^2)$.
- The model allowed correlated district- and division-level random effects to capture dependence between outcome categories $ANC0$ and $ANC4^+$.

Division and District Trends

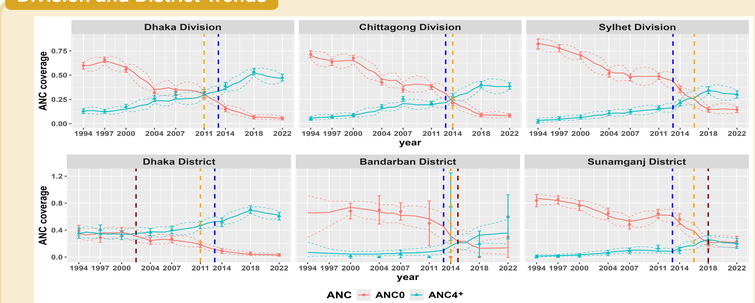

Fig. 4: Selected division and district level trends in the coverage of 0 and 4^+ ANC visits for pregnant women in Bangladesh during 1994-2022. Vertical lines represent National (blue), Division (orange) and District (purple) level achievement.

More Information

Health worker Rashida checking an expecting mother's health condition in Gaibandha, Bangladesh. (C) BRAC /Sumon Yusuf 2018.

Acknowledgments:

- This research was supported by the Australian National University and the National Health and Medical Research Council (NHMRC).
- Special thanks to H. J. Boonstra (`mcmcsae` **R** package) and S. A. Basher (NTL data).

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